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Distinct and Joint Price Approaches for Multi-Layer, Multi-Channel Selling Price by Manufacturer

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Abstract

This article analyses a pricing policy and coordination among members of a three-layer, multi-channel and multi-echelon supply chain, consisting of manufacturers, distributors and retailers. For a single item, demand is assumed to be a linearly decreasing function of time and selling price at the retailer's end. Each supply chain participant earns the largest profit function per unit of time after accounting for expenses. Because the holding cost of commodities is higher in cities than in rural areas, this article proposes a holding cost-sharing idea among wholesalers and merchants. In this article, we maximum retailer initial lot size, selling price, and replenishment time, as well as distributor and manufacturer initial lot size and wholesale pricing. This article is analysed in two frameworks first one is decentralized, and other one is centralized. The optimality conditions of each supply chain member's profit function have been derived with respect to the decision variables and propositions. The results are shown in the data table to illustrate the model. We have also done sensitivity analysis with numerical examples.

Keywords: Distinct; Holding Cost; Multi-Layer Multi-Channel; Selling Price; Supply Chain; Collective Strategies.

1. Introduction

The introductory section consists of two parts. The first part consists of the motivation of research work, whereas the second part is a reported literature review contribution are described.

1.1 Motivation (General Problem Description)

The most captivating and extensively studied subject in production, as well as operation management, is supply chain and inventory. Because it has an impact on our daily lives, inventory is crucial. It may be found in homes, businesses, and social settings alike. Flexibility is offered by inventory, but it is not free. Products accumulated to use to meet future demand are referred to as inventory. The term inventory refers to the physical stock of goods and materials in any business. It includes the stock of goods available for sale as well as the raw materials used to make the goods available for sale. Inventory is a crucial asset for any firm. Its prime objective is to minimize the cost of the firm by maximizing profit. All types of goods or services used by any business organization to sell in the market to earn a profit are called Inventory. However, the assets that are used to earn that profit are not kept in the category of inventory. For example, if an item is produced in a production plant with the help of a machine, then the final product will be

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treated as inventory and the machine used for it is an asset and not inventory. Globalization has developed the complexity of coordination among supply chain members, and recently from the past one-decade globalization has increased exponentially, consequently, the supply chain sector faces major problems. Due to a lack of co-ordination, the needs of consumers are not satisfied over time and therefore the earnings of supply chain members are affected.

1.2 Literature Review:

To tackle these types of problems, increase earnings, and fulfill the customers' needs within time, many models/articles have been designed by many authors and researchers of supply chain management in this research area. In 1994 Parlar and Weng (1994) developed a supply chain inventory model for a single supplier and single retailer under the situation of manufacturer's Stackelberg in which they considered a concept of quality discount scheme. Further, the model of Parlar and Weng (1994) and Singh (2019) which comprised a single supplier and multi distributors under the situation of increasing quantity discount policies with lot size. For the development of an inventory model, a relationship is required between price and demand and a very often a convenient price-demand relationship function is chosen arbitrary, but Lau and Lau (2003) and Gunasekaran and Kobu (2007) suggested a model in which they used different price-demand relationship curve's shape and studied the effects on model output.

Recently, it has been observed that most of the researcher's study two-layer supply chain problems but in real life supply chain networks are more complex and each stage has more supply chain members. Daya et al. (2013) formulated a model for a three-layer supply chain which consisted a one manufacturer, one supplier and multi-retailers. Chan et al. (2017) optimized the production rate for exponential deteriorating single item for an integrated two-layer supply chain model, consisting of a single vendor and a retailer. This study is an advanced mathematical model compared to traditional by considering variable production rate. Mehata et al. (2019) formulated dynamic decision strategies for deteriorating items under price inflation and permissible payment delays. In this article, he adopted an iso-elastic and selling price-dependent demand and optimized retail price replenishment time and finite time horizon. Krapal et al. (2022), Pal et al. (2023), and Fang et al. (2023) have suggested an optimal decision policy for a dual supply chain in a competitive environment considering green supply chain strategies and promotional efforts. Kumar et al. (2023) explained a production inventory model for constant demand rate. Hariom et al. (2024) suggested an inventory model for time-dependent linear demand under three levels of production system considering shortage.

For deterioration many researchers have done work. Pal et al. (2013) formulated the optimum lot size formulas for provider and production rate for manufacturers beneath three stage trade credit financing policy for three-layer supply chain which consisted supplier, manufacturer and retailers. Cardenas- Barron and Sana (2015) formulated two two-layer supply chain inventory model for a promotional efforts cost sensitive demand incorporating a delay of payment is offered by supplier to the retailer. Lin et al. (2022) developed an inventory model for a deteriorating item which is deteriorate in quality and quantity. This study is designed for two stage trade credit policy and optimized the retailer's responses when involving both quality and quantity losses. Singh et al. (2022) designed an inventory model of a supply chain for deteriorating item under selling price. This study optimized retailer's replenishment rate when demand rate is declining with time. In this study they assumed that, sequentially provides a fixed credit period by supplier to the manufacturer and manufacturer provides to the retailer and retailers provide, to the customers.

Song and He (2019) Designed a three-layer supply inventory model for products of the agricultural sector. The study is developed for two different strategies which one are centralized and decentralized and optimized unit online selling price, unit logistics distribution price, fresh-keeping effort. Giri et al. (2021) investigated a co-ordination issue in a three-layer supply chain with single raw material supplier and single manufacturer and single retailer under the demand is not deterministic. Singh et al. (2021) Suggested a co-ordination policy for a production system considering finished products and raw materials under different situations. Zhao and Chen (2023) analyzed the pricing strategies for a two-echelon supply chain inventory model which consisted a single manufacturer and two retailers. The retailing price, sales effort, and order quantity are determined and optimized at retailers ends. The profit functions of both supply chain member are optimized under two different strategies, decentralized and centralized. Xu et al. (2023) suggested a supply chain coordination policy for an online platform under the green channel supply chain technology.

A three-layer multi-channel and multi-echelon supply chain inventory model is formulated by Modak et al. (2016) for a single item. Shaikh et al. (2021) developed two level trade credit policy considering with expiration rate and impact of in their demand under nonzero inventory and partial backlogged. A reverse supply chain coordination policy for multi -collector, multi- distributors and single manufacturers is developed by Nigwal et al. (2022). Singh et al. (2023)

have explained supply chain model, this article they assumed that the supply chain consisted of one manufacturer, multi retailers and distributors as the supply chain members. Seasonal products deteriorate very fast and after sales season these become useless and the deterioration rate is controlled by item preservation technology. Using this concept

Nowadays is it need to reuse the product is going felt, on the basis of this concept Shukla and Khedlekar (2016) designed an inventory model for convertible items in which they consider the item that converts one form to more than one another forms by investing conversion cost and time. Panda et al. (2017) suggested a three-layer echelon supply chain model which consisted a single manufacturer, multi-distributor and multi-retailer. In this study a systematic co-ordination strategy is formed and benefit benefit-sharing contract is made for all supply chain members for deteriorating single products. Shah et al. (2023) developed a closed-loop supply chain inventory model which consisted of one manufacturer and one retailer. In this article, they assumed the manufacturer and retailer both optimize their own profit by product retailing and recycling and play a social responsibility. The study is analysed in two different frameworks first one is centralized and second one is decentralized. In the production system disruption is a common phenomenon in real life. They optimized the total convertible cost and conversion time for deteriorating products by assuming deterioration rates differ at each convertible stage.

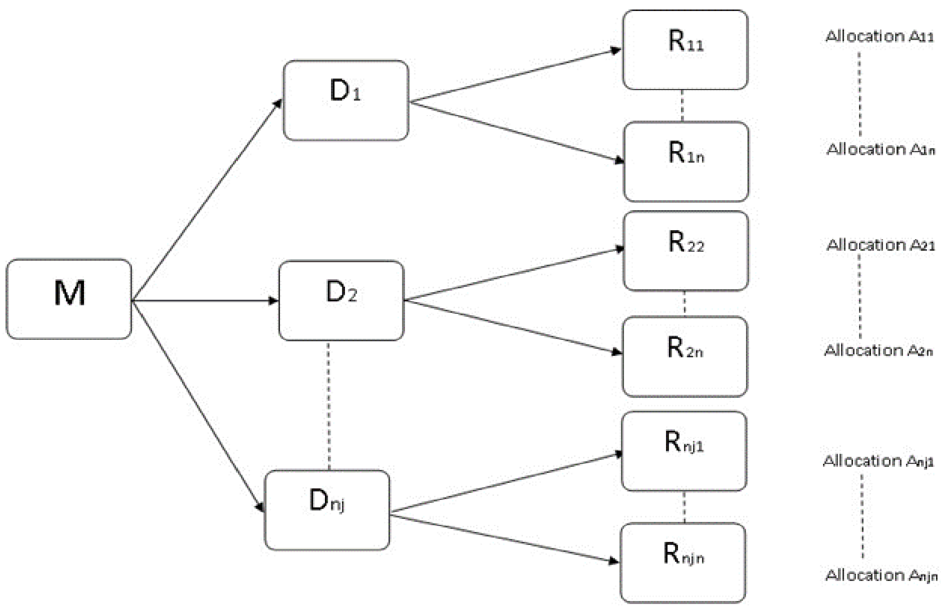

Figure 1. Supply Chain Distribution Network

In this article, we consider a three-layer multi-channel and multi-echelon supply chain in which a single manufacturer occupies the first echelon stage, second stages are occupied by multi distributors and third stages are also occupied by multi-retailers (Fig.1). Firstly, the manufacturer delivers a fixed lot of a single item to the j^{th} $(j = 12 \dots \text{n})$ distributors and j^{th} distributors also provide a single item to the $ij^{th}(i = 123 \dots n_j), (j = 12 \dots . \text{n})$ retailers, where each retailer is associated with to a certain distributor. The total demand all retailers of single item is fulfilled by all distributors and the total demand of all distributors' is fulfilled by the single manufacturer. Manufacturers and distributors obey the EOQ conveyance policy. The finite replenishment time for retailers must be equally applicable for all distributors as well as manufacturers, therefore we will find replenishment time only for retailers.

This study provides a multi-channel and multi-echelon inventory model for decomposing objects that has a consistent, predictable rate of deterioration for all objects and a time-shifting holding cost. Failed attempts at fractional accumulating are acceptable in this test. Nevertheless, a transporter has a cut-off limit, and we are not allowed to assemble an infinite quantity of things. This research aims to find optimal replenishment time, selling price, and initial lot size for retailers in a centralized and decentralized framework under the demand of retailer's end-suggested selling price by the manufacturer of the item. The demand is a linearly declining function of time, and selling price. Furthermore, it has been also assumed that a holding cost is shared among the retailers and distributors.

2. Notations and Assumptions

The notations and presumptions mentioned below form the foundation of the mathematical model.

2.1. Notations

The following are the cost parameters set by the manufacturer:

(i) s^m : Maximum selling price per unit item suggested by the manufacturer.

(ii) d_{ij}^R: ij^{th} retailer's demand per unit time per unit of the item.

(iii) d_j^D: j^{th} distributor's demand per unit time per unit of the item.

(iv) d^M : Manufacture's demand per unit time per unit item.

(v) s_{ij}^R : Selling price per unit item for ij^{th} retailer in a decentralized framework, where $s_{ij}^R > W_j^d$.

(vi) s_{ij}^{RC} : Selling price per unit item for ij^{th} retailer in centralized framework.

(vii) w_j^d: Distributor's wholesale price per unit item, where $w_j^d > w^m$.

(viii) w^m : Manufacturer's wholesale price per unit item, where $w^m > c$.

(ix) c : Production cost per unit item for manufacturer.

(x) z_{ij}^R: ij^{th} retailer's average profit in a decentralized framework.

(xi) z_j^D: j^{th} distributor's average profit in decentralized framework.

(xii) z^M: Manufacturer's average profit in a decentralized framework.

(xiii) n : Number of distributors.

(xiv) n_j : Number of retailers.

(xv) z^c : Average profit of the whole channel in a centralized framework.

(xvi) β : Difference coefficient of s_{ij}^R and s^m, when either $s_{ij}^R \geq s^m$ or $s_{ij}^R \leq s^m$.

(xvii) α : Price sensitive parameter of demand function.

(xviii) T : Finite time horizon.

(xix) $q_{ij}^r(t)$: Initial lot size of ij^{th} retailer's end.

(xx) $q_j^d(t)$: Initial lot size of j^{th} distributor's end.

(xxi) $q^m(t)$: Initial lot size of manufacturer.

(xxii) δ : Holding cost sharing coefficient.

(xxiii) h : Holding cost per unit item per unit time.

2.2. Assumptions

This requirement has been met in order to formulate the issue.

(i) Demand per unit time of the item in the market is $d_{ij}^{R} = a_{ij} - b_{ij}t - \alpha s_{ij}^{R} + \beta\left(s^{m} - s_{ij}^{R}\right)$, is a linear declining function of t, selling price and difference coefficient of suggested selling price and selling price, where a_{ij} is initial demand scale parameter, β is difference coefficient of s^{m} and $s_{ij}^{R}, a_{ij} > 0, b_{ij} > 0, \beta > 0, \alpha > 0$, and $0 \leq t \leq T$.

(ii) Holding costs are constant and shared among distributors and retailers.

(iii) $a_j = \sum_{i=1}^{n_j} a_{ij} b_j = \sum_{i=1}^{n_j} b_{ij}$ and $a = \sum_{i=1}^{n_j}\sum_{j=1}^{n} a_{ij}, b = \sum_{i=1}^{n_j}\sum_{j=1}^{n} b_{ij}$

(iv) There is no competitive environment among second and third echelon stages.

(v) Finite interval time T is evaluated only for retailers and which is equally applicable on whole supply chain.

3. Individual Pricing Strategy

In this framework the manufacturer acts as a Stackelberg leader, distributors and retailers act as followers of manufacturer. Although, the manufacturer is the Stakleberg leader of supply chain, but he cannot determine selling price of the item, he may only suggest a selling price publicly, at which the item is expected to be sold. In common parlance, such selling price is called maximum suggested selling price (MSSP). Because all channel members are located sequentially in the different echelon stage, they can independently decide to optimize their individual goals. As per wholesale price, demand of the item and based on known information, retailers' may take own strategic decision about his goal. Therefore, the retailers' model could be formulated first as follows.

3.1. Retailers' Individual Pricing Strategy

Manufacturer knows well about the specification of their item and related manufacturing expenditure. Therefore, manufacturer determines the maximum selling price at which the item expected to be sold. The maximum suggested selling price is generally printed on packet or tag of the item. Generally, as per the market situations and quality of item, consumers are either satisfied or dissatisfied with manufacturer's determined retail price. Let a manufacturer provides stock of the item to the n distributors $D_{1j}, D_{2j}, D_{3j}, \ldots D_{nj}$. Distributors also provide lots of the item to the $R_{11}, R_{12}, R_{13}, \ldots R_{nn}$ retailers respectively. Let ij^{th} retailer receives the stock, at time $t, t \in [0, \mathrm{T}]$. The rate of change in the ij^{th} retailer's initial lot size q_{ij}^{R} units of item of ij^{th} retailer. At any time t following nonlinear equation represents the inventory status for ij^{th} retailer

$$\frac{dI_{ij}^{R}(t)}{dt} = -d_{ij}^{R}, \text{ where } 0 \leq t \leq T \tag{1}$$

with initial condition $I_{ij}^{R}(t) = 0$, at $t = T$, where $i = 1,2,3, \ldots, n_j$ and $j = 1,2,3, \ldots, n$, Equation (1) yields

$$I_{ij}^{R}(t) = a_{ij}(T - t) - \frac{b_{ij}(T^2 - t^2)}{2} + (\alpha + \beta)s_{ij}^{R}(t - T) + \beta s^{m}(T - t) \tag{2}$$

The initial inventory level $I_{ij}^{R}(0) = q_{ij}^{r}$ for ij^{th} retailer's end at finite time $t = 0$, where $t \in [0, T]$ will be

$$I_{ij}^{R}(0) = q_{ij}^{R} = a_{ij}T - \frac{b_{ij}T^2}{2} - (\alpha + \beta)s_{ij}^{R}T + \beta s^{m}T \tag{3}$$

The total sales revenue SR_{ij}^{r} in the replenishment time period $[0, T]$ could be formulated as

$$SR_{ij}^{R} = \int_0^T s_{ij}^{R} q_{ij}^{R} dt$$

$$SR_{ij}^{R} = s_{ij}^{R}\left(a_{ij}T - \frac{b_{ij}T^2}{2} - (\alpha + \beta)s_{ij}^{R}T + \beta s^{m}T\right) \tag{4}$$

Purchase expenditure PE_{ij}^{R} of ij^{th} retailer is

$$PE_{ij}^{R} = \int_0^T w_j^{d} q_{ij}^{R} dt$$

$$PE_{ij}^{R} = w_j^{d}\left(a_{ij}T - \frac{b_{ij}T^2}{2} - (\alpha + \beta)s_{ij}^{R}T + \beta s^{m}T\right) \tag{5}$$

The inventory holding expenditure IHE_{ij}^{R} of ij^{th} retailer is

$$IHE_{ij}^{R} = h\int_0^T I_{ij}^{R}(t)dt$$

$$IHE_{ij}^{R} = h\left(\frac{a_{ij}T^2}{2} - \frac{b_{ij}T^3}{3} - (\alpha+\beta)s_{ij}^{R}\frac{T^2}{2} + \beta s^m\frac{T^2}{2}\right) \quad (6)$$

Hence, the average profit function per unit time is

$$z_{ij}^{R} = \begin{bmatrix} \left(s_{ij}^{R} - w_j^{d}\right)\left\{a_{ij} - \frac{b_{ij}T}{2} - (\alpha+\beta)s_{ij}^{R} + \beta s^m\right\} \\ -h\delta\left(\frac{a_{ij}T}{2} - \frac{b_{ij}T^2}{3} - (\alpha+\beta)s_{ij}^{R}\frac{T}{2} + \beta s^m\frac{T}{2}\right) \end{bmatrix} \quad (7)$$

Proposition 3.1 If the demand of items is uniformly at all retailer's end with respect to time T and $s_{ij}^{R} > w_j^{d}(i = 1,2,3 \ldots m$ and $j = 1,2,3 \ldots n)$. Then the ij^{th} retailer's profit function shows concavity in selling price s_{ij}^{R} and replenishment time T, if $\frac{4}{3}(\alpha+\beta)h\delta b_{ij} - \frac{1}{4}\left(\delta h(\alpha+\beta) - b_{ij}\right)^2 > 0$.

Proof: The second order partial derivative of ij^{th} retailer's profit function z_{ij}^{R} with respect to s_{ij}^{R} and T respectively are

$$\frac{\partial^2 z_{ij}^{R}}{\partial s_{ij}^{R2}} = -2(\alpha+\beta)$$

$$\frac{\partial^2 z_{ij}^{R}}{\partial s_{ij}^{R}\,\partial T} = \frac{h\delta(\alpha+\beta) - b_{ij}}{2}$$

$$\frac{\partial^2 z_{ij}^{R}}{\partial T^2} = -\frac{2b_{ij}h\delta}{3}$$

Retailer's profit function z_{ij}^{R} shows concavity with respect to s_{ij}^{R} and T, if the Hessian matrix of z_{ij}^{r}, is negative semi definite

$$HM = \begin{bmatrix} \frac{\partial^2 z_{ij}^{R}}{\partial s_{ij}^{R2}} & \frac{\partial^2 z_{ij}^{R}}{\partial s_{ij}^{R}\,\partial T} \\ \frac{\partial^2 z_{ij}^{R}}{\partial s_{ij}^{R}\,\partial T} & \frac{\partial^2 z_{ij}^{R}}{\partial T^2} \end{bmatrix} = \begin{bmatrix} -2(\alpha+\beta) & \frac{h\delta(\alpha+\beta)-b_{ij}}{2} \\ \frac{h\delta(\alpha+\beta)-b_{ij}}{2} & -\frac{2b_{ij}h\lambda}{3} \end{bmatrix} \quad (8)$$

Expansion of Hessian matrix gives $16(\alpha+\beta)b_{ij}h\delta - 3\left(\delta h(\alpha+\beta) - b_{ij}\right)^2 > 0.$, if $\beta > 0, \alpha > 0$ and hence Hessian matrix of z_{ij}^{R}, is negative semi definite in s_{ij}^{R} and T if $16(\alpha+\beta)b_{ij}h\delta - 3\left(\delta h(\alpha+\beta) - b_{ij}\right)^2 > 0$.

Proposition 3.2. If the demand is uniformly decreasing function of suggested price and time at all ij^{th} retailer's end and $s_{ij}^{R} > w_j^{d}(i = 1,2,3 \ldots m$ and $j = 1,2,3 \ldots n)$. Then optimum selling price s_{ij}^{R} is given by the equation (9) and optimum replenishment time T can be find by satisfying the equation (10).

$$s_{ij}^{R} = \frac{w_j^{d}}{2} + \frac{\beta s^m}{2(\alpha+\beta)} + \frac{\delta hT}{4} + \frac{2a_{ij} - b_{ij}T}{4(\alpha+\beta)} \quad (9)$$

$$\left(s_{ij}^{R} - w_j^{d}\right)b_{ij} + \delta h\left(a_{ij} - (\alpha+\beta)b_{ij} + \beta s^m\right) - \frac{4}{3}b_{ij}\delta hT = 0. \quad (10)$$

Proof: Equate to zero the first order partial derivatives of equation (7), yields

$$(\alpha+\beta)\left(s_{ij}^{R} - w_j^{d}\right) - \frac{2a_{ij} - b_{ij}T}{2} + (\alpha+\beta)s_{ij}^{R} - \beta s^m + h\delta(\alpha+\beta)\frac{T}{2} = 0 \quad (11)$$

$$\frac{b_{ij}\left(s_{ij}^{R} - w_j^{d}\right)}{2} + \delta h\left[\frac{a_{ij}}{2} - \frac{2b_{ij}T}{3} - \frac{(\alpha+\beta)s_{ij}^{R}}{2} + \frac{\beta s^m}{2}\right] = 0 \quad (12)$$

Solution of these simultaneous equation gives the required results.

3.2. Distributors' Individual Pricing Strategy

Let $D_{1j}, D_{2j}, D_{3j}, \ldots D_{nj}$ be the n^{th} distributors and the demand of distributors end is a sum of all respective retailer's demand.

Corollary 3.1 If the coefficient b_{ij} is uniformly distributive with respect to time and $a_j = \sum_{i=1}^{n_j} a_{ij}\ b_j = \sum_{i=1}^{n_j} b_{ij}$, then the demand of items at d_j^D distributors' end can be determined by the following formula

$$d_j^D = \sum_{i=1}^{n_j} d_{ij}^R = a_j - b_j t - (\alpha+\beta)\sum_{j=1}^{n_j} s_{ij}^R + n_j \beta s^m \quad (13)$$

Now at the time t the rate of changes in the j^{th} distributor's inventory level can be balanced by the sum of all associated retailers' demand which are affiliated to the j^{th} distributor. Therefore, at any time t, j^{th} distributor's inventory can be represented by the equation

$$\frac{dI_j^D(t)}{dt} = -d_j^D \text{ where } 0 \le t \le \quad (14)$$

with initial condition $I_j^D(t) = 0$, at $t = T$, where $j = 1,2,3, \ldots, n$, Equation (14) yields

$$I_j^D(t) = a_j(T-t) - \frac{b_j}{2}(T^2 - t^2) + (\alpha+\beta)\sum_{j=1}^{n_j} s_{ij}^R (t-T) + \beta s^m n_j (T-t) \quad (15)$$

The initial lot size for j^{th} retailer at any time $t = 0$, where $t \in [0, T]$ will be

$$I_j^D(0) = q_j^D = a_j T - \frac{b_j T^2}{2} - (\alpha+\beta)\sum_{j=1}^{n_j} s_{ij}^R T + \beta p^m n_j T \quad (16)$$

The sales revenue of j^{th} distributor SR_j^D in the finite time interval $[0, T]$ can be find as

$$SR_j^D = \int_0^T w_j^d q_j^D dt$$
$$SR_j^D = w_j^d\left(a_j T - \frac{b_j T^2}{2} - (\alpha+\beta)\sum_{j=1}^{n_j} s_{ij}^R T + \beta s^m T \eta_j\right) \quad (17)$$

Purchase expenditure of j^{th} distributor is

$$PE_j^D = \int_0^T w^m d_j^D dt = w^m\left(a_j T - \frac{b_j T^2}{2} - (\alpha+\beta)\sum_{j=1}^{n_j} s_{ij}^R T + \beta s^m T \eta_j\right) \quad (18)$$

The inventory holding expenditure IHC_j^d for j^{th} distributor is

$$IHE_j^D = h\int_0^T I_j^D(t)dt == h\left(\frac{a_j T^2}{2} - \frac{b_j T^3}{6} - (\alpha+\beta)\sum_{i=1}^{n_j} \frac{p_{ij}^r T^2}{2} + \frac{\beta s^m n_j T^2}{2}\right) \quad (19)$$

Hence the average profit function per unit time for j^{th} distributor is

$$z_j^D = \begin{bmatrix} (w_j^d - w^m)\left\{a_j - \frac{b_j T}{2} - (\alpha+\beta)\sum_{j=1}^{n_j} s_{ij}^R + \beta s^m \eta_j\right\} \\ -h(1-\delta)\left(\frac{a_j T}{2} - \frac{b_j T^2}{6} - (\alpha+\beta)\sum_{i=1}^{n_j} \frac{s_{ij}^R T}{2} + \frac{\beta s^m n_j T}{2}\right) \end{bmatrix} \quad (20)$$

Proposition 3.3 If the demand is uniformly decreasing function of suggested price and time T at all j^{th} distributors' end and $w_j^d > w^m (j = 1,2,3 \ldots n)$. Then the optimum whole sale price w_j^d is given by the equation (21).

$$w_j^d = \left[\frac{w^m}{2} + \frac{2a_j - b_j T}{4(\alpha+\beta)n_j} - \frac{\delta h T}{4} - \frac{\beta s^m}{2(\alpha+\beta)} + \frac{h(1-\delta)}{2}\right] \quad (21)$$

Proof: Partial differentiation of equation (20) with respect to w_j^d yields

$$\frac{\partial z_j^d}{\partial d_j^d} = \begin{bmatrix} -\frac{(w_j^d - w^m)n_j(\alpha+\beta)}{2} + \frac{2a_j - b_j T}{4} - \frac{w_j^d n_j(\alpha+\beta)}{2} \\ -\frac{\beta s^m n_j}{2} - \frac{(\alpha+\beta)h\delta n_j T}{4} + \frac{h(1-\delta)n_j(\alpha+\beta)}{2} \end{bmatrix}$$

At the optimum value of $w_j^d, \frac{\partial z_j^d}{\partial w_j^d} = 0$ i.e.

$$\left[\begin{array}{l} -\frac{\left(w_j^d - w^m\right)n_j(\alpha+\beta)}{2} + \frac{2a_j - b_jT}{4} - \frac{w_j^d n_j(\alpha+\beta)}{2} \\ -\frac{\beta s^m n_j}{2} - \frac{(\alpha+\beta)h\delta n_j T}{4} + \frac{h(1-\delta)n_j(\alpha+\beta)}{2} \end{array}\right] = 0 \tag{22}$$

Solution of equation (22), yields

$$w_j^d = \frac{w^m}{2} + \frac{2a_j - b_jT}{4(\alpha+\beta)n_j} - \frac{\delta hT}{4} - \frac{\beta s^m}{2(\alpha+\beta)} + \frac{h(1-\delta)}{2} \tag{23}$$

Also, profit function z_j^d shows optimality with respect to w_j^d, because

$$\frac{\partial^2 z_j^d}{\partial {w_j^d}^2} = -n_j(\alpha+\beta) \tag{24}$$

for, if $\beta > 0$ and $\alpha > 0$.

3.3. Manufacturer's Individual Pricing Strategy

Manufacturer provides an initial lot size of items to all distributors as per their demands.

Corollary 3.2 If the coefficient b_j is uniformly distributive with respect to T and $a = \sum_{i=1}^{n_j}\sum_{j=1}^{n} a_{ij}$ $b = \sum_{i=1}^{n_j}\sum_{j=1}^{n} b_{ij}$, then the demand of items at manufacturer's end can be determined by the following formula

$$d^M = \sum_{j=1}^{n} d_j^D = a - bt - (\alpha+\beta)\sum_{i=1}^{n_j}\sum_{j=1}^{n} s_{ij}^R + m\beta s^m \tag{25}$$

The rate of changes in the manufacturer's inventory level is balanced by all j^{th} distributor's demand. At any movement t manufacturer's inventory level can be represented as

$$\frac{dI^m(t)}{dt} = -d^M \text{, where } 0 \le t \le T \tag{26}$$

with boundary condition $I^m(t) = 0$, at $t = T$. Solution of equation (26) yields

$$I^m(t) = a(T-t) - \frac{b\left(T^2 - t^2\right)}{2} + (\alpha+\beta)\sum_{i=1}^{n_j}\sum_{j=1}^{n} s_{ij}^R(t-T) + \beta s^m m(T-t) \tag{27}$$

The initial lot size for manufacturer at time $t = 0$, where $t \in [0, T]$ is

$$I^m(0) = q^m = aT - \frac{bT^2}{2} - (\alpha+\beta)\sum_{i=1}^{n_j}\sum_{j=1}^{n} s_{ij}^R T + \beta s^m mT \tag{28}$$

The sales revenue in the finite time $[0, T]$ can be determined as

$$SR^m = \int_0^T w^m d^M dt = w^m\left(aT - \frac{bT^2}{2} - (\alpha+\beta)\sum_{i=1}^{n_j}\sum_{j=1}^{n} s_{ij}^R T + \beta s^m Tm\right) \tag{29}$$

Manufacturing expenditure for manufacturer is

$$MC^m = c\int_0^T d^M dt = c\left(aT - \frac{bT^2}{2} - (\alpha+\beta)\sum_{i=1}^{n_k}\sum_{j=1}^{n} s_{ij}^R T + \beta p_m Tm\right) \tag{30}$$

Hence the average profit function z^m per unit time for manufacturer is

$$z^m = (w^m - c)\left[a - \frac{bT}{2} - (\alpha+\beta)\sum_{i=1}^{n_j}\sum_{j=1}^{n} s_{ij}^R + \beta s^m m\right] \tag{31}$$

Proposition 3.4 If the demand is uniformly decreasing function of suggested price and time T at manufacturer's end and $w^m > c$. Then the optimum whole sale price w^m is given by the equation (32).

$$w^m = \frac{c}{2} + \frac{2a - bT}{4(\alpha+\beta)m} + \frac{3\beta s^m}{2(\alpha+\beta)} - \frac{\delta hT}{4} - \frac{h(1-\delta)}{2(\alpha+\beta)} \tag{32}$$

Proof: Partial differentiation of equation (31) with respect to w^m yields

$$\frac{\partial z^m}{\partial w^m}=\begin{bmatrix} -\frac{(w^m-c)(\alpha+\beta)m}{4}+a-\frac{bT}{2}-\frac{(\alpha+\beta)w^m m}{4}-\frac{2a-bT}{8} \\ -\frac{m\beta s^m}{4}-\frac{\delta(\alpha+\beta)hmT}{8}-\frac{2a-bT}{4}-\frac{(1-\delta)hm}{4}+m\beta s^m \end{bmatrix} \quad (33)$$

At the optimum value of $w^m, \frac{\partial z^m}{\partial w^m}=0$, i.e.

$$\begin{bmatrix} -\frac{(w^m-c)(\alpha+\beta)m}{4}+a-\frac{bT}{2}-\frac{(\alpha+\beta)w^m m}{4}-\frac{2a-bT}{8} \\ -\frac{m\beta s^m}{4}-\frac{\delta(\alpha+\beta)hmT}{8}-\frac{2a-bT}{4}-\frac{(1-\delta)hm}{4}+m\beta s^m \end{bmatrix}=0 \quad (34)$$

Solution of the equation (34) yields (32) Also profit function z^m shows optimality with respect to w^m, we have

$$\frac{\partial^2 z_j^d}{\partial w^{m^2}}=\frac{-n(\alpha+\beta)}{2}$$

for if $\beta>0$ and $\alpha>0$.

4. Collective / Joint Pricing Strategy

In this pricing strategy, the whole supply chain members work together as a single unit and all the members of supply chain cooperate perfectly to each other to maximize the performance of supply chain. The manufacturer is a leader of whole supply chain as a single decision maker and all decisions are equally applicable to the whole supply chain members. Therefore, for optimization of whole channel's profit he can take all decisions.

4.1. Given Model for Proposed

Let s_{ij}^{RC} is a selling price of ij^{th} retailer, w_j^d is a whole sale price of j^{th} distributor, w^m is a whole sale price of manufacturer, c is the manufacturing cost, IHC_{ij}^r is the holding cost of ij^{th} retailer and IHC_j^d is holding cost of j^{th} distributor, then the profit function is

$$Z^c=\sum_{i=1}^{m}\sum_{j=1}^{m_j}\left[\left(s_{ij}^{RC}-w_j^d\right)d_{ij}^R-\delta\left(IHC_{ij}^R\right)\right]$$
$$+\sum_{i=1}^{n}\left[\left(w_j^d-w^m\right)d_j^D-(1-\lambda)IHC_j^D\right]+(w^m-c)d^m$$
$$Z^c=\sum_{i=1}^{n_j}\sum_{j=1}^{n}\left[\left(p_{ij}^r-c\right)d_{ij}^R-\left(IHC_{ij}^R\right)\right]$$

Hence the average profit function z^c per unit time is

$$z^c=\begin{bmatrix} \sum_{i=1}^{n_j}\sum_{j=1}^{n}\left(s_{ij}^R-c\right)\left(a_{ij}-\frac{b_{ij}T}{2}-(\alpha+\beta)s_{ij}^R+\beta s^m\right) \\ -\sum_{i=1}^{m_j}\sum_{j=1}^{m} h\left(\frac{a_{ij}T}{2}-\frac{b_{ij}T^2}{3}-\frac{(\alpha+\beta)s_{ij}^R T}{2}+\frac{\beta s^m T}{2}\right) \end{bmatrix} \quad (35)$$

Proposition 4.1. If the demand of items is uniformly at all retailer's end with respect to time T and $s_{ij}^{RC}>c(i=1,2,3\ldots m$ and $j=1,2,3\ldots n)$. Then the whole supply chain profit function shows concavity in selling price s_{ij}^{RC} and replenishment time T, if $-\frac{4}{3}(\alpha+\beta)hbm-\frac{1}{4}(h(\alpha+\beta)m-b)^2>0$.

Proof: Using equation (35), the second order partial derivatives in selling s_{jk}^{RC}, and time T of the profit function respectively are

$$\frac{\partial^2 z^c}{\partial s_{ij}^{RC^2}}=-2m(\alpha+\beta) \quad (36)$$

$$\frac{\partial^2 z^c}{\partial s_{ij}^{RC}\partial T} = \left(\frac{h(\alpha+\beta)m}{2} - \frac{b}{2}\right) \quad (37)$$

$$\frac{\partial^2 z^c}{\partial T^2} = \frac{2bh}{3} \quad (38)$$

the profit function z^c must be jointly concave with respect to s_{ij}^{RC} and T, if the Hessian matrix of profit function z^c, is negative semi definite

$$HM = \begin{bmatrix} \frac{\partial^2 z^c}{\partial s_{ij}^{rc2}} & \frac{\partial^2 z^c}{\partial p_{ij}^{RC}\partial T} \\ \frac{\partial^2 z^c}{\partial p_{ij}^{RC}\partial T} & \frac{\partial^2 z^c}{\partial T^2} \end{bmatrix} = \begin{bmatrix} \frac{2bh}{3} & \left(\frac{h(\alpha+\beta)m}{2} - \frac{b}{2}\right) \\ \left(\frac{h(\alpha+\beta)m}{2} - \frac{b}{2}\right) & -2m(\alpha+\beta) \end{bmatrix} \quad (39)$$

Hence, if $-\frac{4}{3}(\alpha+\beta)hbm - \frac{1}{4}(h(\alpha+\beta)m - b)^2 > 0.$, then the Hessian matrix of the profit π^c, must be negative semi definite and thus the profit function π^c is jointly concave in p_{ij}^{rc} and T. Hence proved it.

Proposition 4.2 If the demand is uniformly decreasing function of suggested price and time T at all ij^{th} retailer's end and $s_{ij}^{RC} > c(i = 1,2,3 \dots m$ and $j = 1,2,3 \dots n)$. Then optimum selling price s_{ij}^{RC} is given by the equation (40) and optimum replenishment time T can be find by satisfying the equation (41).

$$s_{ij}^{RC} = \frac{c}{2} + \frac{2a_{ij} - b_{ij}T}{4(\alpha+\beta)} + \frac{\beta s^m}{2(\alpha+\beta)} + \frac{hT}{4} \quad (40)$$

and

$$\frac{s_{ij}^{RC} b_{ij}}{2} - \frac{cb_{ij}}{2} + \frac{a_{ij}h}{2} - \frac{2b_{ij}Th}{3} - \frac{(\alpha+\beta)hs_{ij}^{RC}}{2} + \frac{\beta s^m h}{2} = 0 \quad (41)$$

Proof: Equate to zero the first order partial derivatives of equation (35), yields

$$\frac{\partial z^c}{\partial s_{ij}^{RC}} = \begin{bmatrix} \sum_{i=1}^{n_j} \sum_{j=1}^{n} \left(s_{ij}^{R} - c\right)\left(a_{ij} - \frac{b_{ij}T}{2} - (\alpha+\beta)s_{ij}^{R} + \beta s^m\right) \\ \{-(\alpha+\beta)\} + \sum_{i=1}^{n_j} \sum_{j=1}^{n} h(\alpha+\beta)\frac{T}{2} \end{bmatrix} \quad (42)$$

$$\frac{\partial z^c}{\partial \mathrm{T}} = \left[\sum_{i=1}^{n_j} \sum_{j=1}^{n} \left(s_{ij}^{R} - c\right)\left(-\frac{b_{ij}}{2}\right) - \sum_{i=1}^{m_j} \sum_{j=1}^{n} h\left(\frac{a_{ij}}{2} - \frac{2b_{ij}T}{3} - \frac{(\alpha+\beta)s_{ij}^{R}}{2} + \frac{\beta s^m}{2}\right)\right] = 0 \quad (43)$$

Solution of these simultaneous gives the required results.

5. Numerical Example

For illustration of this model, we considered a three-layer echelon supply chain which consists a single manufacturer M, two distributors (D_1, D_2)) and four retailers (R_{11}, R_{12}, R_{21} and R_{22}) respectively at each echelon stages. As per Fig.1, each retailer is associated with particular distributor. We consider the following data set for individual and collective pricing strategies, the demand scale parameters are a_{11} = 1588, a_{12} =1585, $a_{21} = 1590$, a_{22} = 1581, b_{11} = 0:01, $b_{12} = 0.01$, $b_{21} = 0.01$, $b_{22} = 0.01$ units, suggested selling price $s^m = 1025$, price sensitive $\alpha = 0.9$, $\delta = 0.01$, $h = 0.03$, $s_c = 50$, coefficient of $\beta = 0.15$ and manufacturing cost is $c = 950$. The model's optimum outputs are shown in the following tables.

Table 1. Individual Pricing Strategy

Optimal	R_{11}	R_{12}	R_{13}	R_{14}	D_1	D_2	M
Price	1507.27	1506.16	1508.31	1504.01	1357.12	1356.6	1352.35
Time	425	420	430	428	-	-	-
EOQ	66721.22	64231.19	61889.01	65651.52	-	-	-
Profit	23471.22	23203.6	22143.59	22508.25	616724.9	580804.4	2160082.95

Table 2. Collective Pricing Strategies

Optimal	R_{11}	R_{12}	R_{13}	R_{14}	D_1	D_2	M
Price	1277.08	1268.62	1271.08	1266.78	-	-	-
Time	455	451	445	449	-	-	-
EOQ	180645	177978	185113	181077	-	-	-
Profit	-	-	-	-	-	-	507616

5.1. Sensitivity Analysis

Corollary 5.1 ij^{th} retailer's profit function is strictly increasing function for the coefficient β i.e., $\frac{\partial z_{ij}^{R}}{\partial \beta} > 0$, if $\left(\left(s_{ij}^{R} - w_{j}^{D}\right) - \frac{h\delta T}{2} < 0\right)$ and $\left(s^{m} - s_{ij}^{R}\right) < 0$.

ij^{th} retailer's profit function shows positive behaviour with respect to the increment of the coefficient β, if $\delta, h, T, s_{i}^{R}, w_{j}^{D}$ must follow the identity $\frac{h\delta T}{2} > \left(s_{m} - w_{j}^{D}\right)$, otherwise increment of the coefficient β, impacted negatively on the ij^{th} retailer's profit function. If δ, h are constants then T can be taken in the interval $T \in \left(\frac{3\left(s^{m} - w_{j}^{d}\right)}{\delta h}, \infty\right)$.

Corollary 5.2 ij^{th} retailer's profit function is strictly decreasing function for the coefficient α, i.e. $\frac{\partial z_{ij}^{R}}{\partial \alpha} < 0$, if $\left(s_{m} - s_{ij}^{R}\right) < 0$ and $\left(\frac{h\delta T}{2} - 1\right) >$.

ij^{th} retailer's profit function shows negative behaviour with respect to the increment of the coefficient α, if $\delta, h, T, s_{i}^{R}, w_{j}^{D}$ must follow the identity $\left(s_{m} - s_{ij}^{R}\right) < 0$ and $\left(\frac{h\delta T}{2} - 1\right) > 0$.

It reveals that the suggested selling price by manufacturer is always less than the ij^{th} retailer's

selling price.

As per the above discussion, the following conclusions are drawn

a. The suggested selling price can be helpful to increase the selling price if it can be kept always less than retailers' selling price and greater than wholesale price.

b. The reorder frequency could be increased to increase the business, if the suggested selling price would be equal to wholesale price of distributors.

c. We can protect the profit function from the negative influence of alpha if we follow the

identities $\left(s^{m} - s_{ij}^{R}\right) < 0$ and $\left(\frac{h\delta T}{2} - 1\right) > 0$.

6. Conclusion

We have designed a three-layer coordinated multi-channel echelon supply chain for two different pricing strategies: the first one is individual, and the second one is collective. This article aims to decide which pricing strategy performs better for all supply chain members in a non-competitive environment. We have found an optimal profit for all supply chain members by using selling price and finite replenishment time for retailer's end as decision variable in both individual and collective strategies. We have also optimized initial lot size for retailer's wholesale prices for manufacturers and distributors. The suggestions are given by in forms of propositions and numerical examples. This article theoretical and practical contribution is how to make co-ordination strategies among multi-echelon supply chain members time-dependent linear decreasing demand. This article is recommended for the inventory manager of supply chain to make a contract to share total profit among manufacturers, distributors and retailers better outcomes. Study concluded that an individual pricing strategy performing better than collective strategy, therefore it is beneficial for managerial purpose in practice. Based on sensitivity analysis, the manufacturer should maintain the suggested selling price less than actual selling price.

The future research scope of this article can be extended by using competitive environment among echelon members. One can be extended this model by incorporating trade credit financing scheme. Model may be also extended by incorporating marketing efforts at retailers' end. It can also be extended by incorporating promotional cost sharing among manufacturers and retailers or manufacturers and distributors.

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