



Web Metaheuristic Algorithm for Capacitated Vehicle Routing Problem

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Abstract

In the business world, a high percentage of prime costs is related to transportation. Any actions to improve transport ways and eliminate unnecessary trips or create alternative shorter routes leads to significant savings in total costs. One of the well-known optimization issues in this regard is capacitated vehicle routing problem (CVRP) that includes arranging vehicle routes while considering its capacity. This problem is among the NP-Hard problems and many different metaheuristic algorithms have been applied for finding its solution, especially in large dimensions. The purpose of this paper is to design a new metaheuristic algorithm, inspired by spiders routing and hunting in cobweb, based on the problem structure which can be used to obtain optimum and near optimum results. Solving three standard problems of CVRP with different dimensions, called P-n19-k2, E-n33-k14, and B-n78-k10 demonstrates that the error values of the proposed algorithm from the optimum answers are less than one percent. Therefore, web metaheuristic algorithm is capable of achieving proper answers in a reasonable time.

Keywords: Capacitated Vehicle Routing Problem; Metaheuristic Algorithm; Combinatorial Optimization.

1. Introduction

Vehicle routing problem (VRP) is a general title attributed to all problems that aim to find optimized routes and minimize vehicle costs in order to meet special demands of customers. This problem, introduced for the first time by Dantzig & Ramser (1959), has an important role in logistic management and goods distribution. VRP is a famous example of integer programming problems, which is among NP-Hard problems and the required calculations for solving this problem increases with scaling according to the problem size. For such problems, it is often desirable to find the approximate answers. It is sufficient to only find the required answers with adequate speed or accuracy in order to obtain the predetermined objectives. This is usually achieved by using different heuristic methods, which are based on insights about the nature of the problem.

VRP is naturally derived from the main problem in scope of transportation, distribution supplies, and logistics (Dantzig & Ramser, 1959). VRP in the real life usually encounters many sub constraints. One of these important limitations is capacitated vehicle routing problem (CVRP) (Li et al., 2005; Ralphs et al., 2003). The purpose of CVRP is to determine routes that minimize total distances or costs, so that the following limitations are considered:

1. Each customer is served exactly once and only in one route.
2. Each vehicle starts and finishes its route from depot.
3. Total demand in each route should not exceed capacity of vehicle.

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Many common techniques are available for solving the vehicle routing problem. These techniques mostly include heuristic and metaheuristic methods because exact algorithms are not able to ensure finding the optimal routes during a reasonable computation, especially when there are several number of cities which make the problem to be an NP-Hard one. Quality of answers produced by metaheuristic methods is higher than classic heuristic methods. Among the metaheuristics, the famous algorithms include Ant Colony Optimization (ACO) (Bullnheimer et al., 1997; Dorigo, 1992; Reimann et al., 2004), Genetic Algorithm (GA) (Berger & Barkaoui, 2003; Holland, 1975), Simulated Annealing (SA) (Arbelaitz et al., 2001), Tabu Search (TS) (Amberg et al., 2000; Xu & Kelly, 1996), and Particle Swarm Optimization (PSO) (Mahmudy et al., 2024). Lin et al. (2009) utilized a hybrid metaheuristic algorithm of SA and TS to solve the CVRP, as one of the most critical problems in optimizing distribution networks, and showed its competitiveness with other existing algorithms used for solving CVRP. (Wang & Lu, 2009) applied a hybrid GA to solve a CVRP. They confirmed the effectiveness of their proposed algorithm by utilizing benchmark problems with less than 100 distribution centers and showed its practicability in optimizing the CVRP by considering a real case. (Miao et al., 2012) addressed a Three-Dimensional Loading CVRP (3L-CVRP) and solved it with a hybrid GA and TS (GATS). Evaluating the proposed algorithm on available test samples revealed its better performance in several cases. Ke & Feng (2013) proposed a two-phase metaheuristic for solving CVRP and investigated its effectiveness empirically. Each iteration of their proposed algorithm consists of two interdependent phases: the first phase determines the potential customers, and the second phase tries to minimize each route's cumulative time. (Caccetta & Abdulniby, 2013) proposed a hybrid algorithm for solving the CVRP. The results of solving ten benchmark instances demonstrated that applying a combination of domain reduction with Clarke and Wright algorithm for solving large size instances of CVRP is far better than using the latter algorithm alone. Tlili et al. (2014) studied the capacitated vehicle routing problem with distance constraints (DCVRP), in which besides having weight limitation, the set of vehicles have distance restrictions. They solved the formulated model with a hybrid metaheuristic algorithm, including particle swarm optimization (PSO) and Variable Neighborhood Search (VNS), and showed that PSO-VNS algorithm is highly competitive in solving the CVRP. Simsir & Ekmekci (2019) considered vehicle routing problem with simultaneous delivery and pickup (VRPSDP) and applied the Artificial Bee Colony (ABC) algorithm to propose the solution. Solving the common benchmark problem revealed that this metaheuristic algorithm is effective in producing very close solutions to the most successful ones in previous studies. With the objective of keeping the populations' diversity and preventing the algorithm from being trapped in local optima, (Altabeeb et al., 2021) proposed a cooperative hybrid firefly algorithm to solve the capacitated vehicle routing problem (CVRP-CHFA). The results of solving 108 examples from 8 benchmarks revealed promising results. Sbai et al. (2022) formulated the postal distribution problem as a CVRP and developed a VNS-GA hybrid algorithm to minimize the total transportation cost. Comparing the proposed algorithm to the other existing approaches clarified its competitiveness in obtaining the solutions. (Jiang et al., 2022) proposed a relevance matrix based evolutionary algorithm, named RMEA, to solve CVRP. The experimental results showed better performance than some of the other algorithms for solving this problem.

Aydinalp & Özgen (2023) presented a mixed-integer programming (MIP) model to discuss the VRP with time windows for a Turkish pharmaceutical company. Comparing the results of two metaheuristic algorithms (SA and adaptive large neighborhood search (ALNS)) clarifies that in small datasets, both algorithms obtain very close values, while the latter outperforms as the dataset becomes larger. To tackle CVRP as an NP-hard problem with high time complexity, (Kumari et al., 2023) developed a novel hybrid metaheuristic algorithm, named GA-RR, by hybridizing GA with ruin and recreate (RR) algorithm. Muriyatmoko et al. (2024) compared the effectiveness of different heuristic and metaheuristic approaches in solving a CVRP of faculty transportation at Universitas Darussalam Gontor. The results of this study showed that metaheuristic algorithms perform better than heuristic approaches for complex case studies. (Arifuddin et al., 2024) solved the CVRP by applying Notndominated-Sorting Genetic Algorithm II (NSGA II) and Grey Wolf Optimizer (GWO). Comparing the performance of these two algorithms revealed slightly better performance of the latter on three datasets, while the computation time had increased.

As reviewing the literature clarifies, various heuristic and metaheuristic methods have been proposed to find the best possible answers of a CVRP. Moreover, the researchers have identified the importance of metaheuristic algorithms in finding appropriate solutions for large scales of this problem, in a reasonable time. Considering the routing and hunting method of spiders in cobweb, this research aims to develop a new metaheuristic algorithm for getting optimum and near optimum results. The performance of the proposed algorithm is then evaluated by solving three standard problems of CVRP and the results show that the generated answers by new developed Web Metaheuristic Algorithm are only

one percent away from the optimal points. Thus, it can be applied for obtaining proper results in a reasonable time. The rest of the paper is organized as follows. Section 2 introduces capacitated vehicle routing problem. Modeling of problem based on simulation of spiders' behavior is proved in section 3. Description of suggested algorithm is an issue discussed in section 4. Web algorithm is a title that will be used for suggested Metaheuristic. A number of standard examples are solved in section 5 by web algorithm and will be considered as an analysis of obtained answers. Finally, section 6 provides the results and suggestions.

2. Capacitated vehicle routing problem

CVRP is a vehicle routing problem in which a fixed number of capacitated vehicles should provide services to the customers. Each vehicle starts its motion from a depot and comes back there with the least costs. The objective function of this problem is minimization of the total distance traveled by the vehicles. This is a possible answer that the total amounts allocated to each route should not be more than the vehicles' capacity which serve the path.

CVRP is a weighted graph $G(V, E)$ in which $V = \{v_0, v_1, \dots, v_n\}$ represents the collection of nodes and $E = \{(v_i, v_j) / v_i, v_j \in V; i \neq j\}$ is the collection of graph edges. v_0 Denotes depot and other nodes denote number of N customers that should be serviced. For each node, a fixed amount of q_i of required demand of customers is dedicated. ($q_0 = 0$ is allocated to v_0 depot). For each edge (v_i, v_j) , there is an amount of d_{ij} which indicates the distance between two cities of v_i and v_j . Each route starts from v_0 which finally leads to this depot. Each v_i city should be visited one time and the amount of distributed goods in one route should not exceed the capacity (Q).

Therefore, the CVRP can be formulated as follows:

$$\text{Minimize } \sum_{k=1}^K \sum_{i=0}^N \sum_{j=0}^N c_{ij}^k x_{ij}^k \quad (1)$$

Subject to:

$$x_{ij}^k = \begin{cases} 1 & \text{if vehicle } k \text{ travels from customer } v_i \text{ to } v_j \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

$$\sum_{k=1}^K \sum_{i=0}^N x_{ij}^k = 1, \quad j = 1, 2, \dots, N, \quad (3)$$

$$\sum_{k=1}^K \sum_{j=0}^N x_{ij}^k = 1, \quad i = 1, 2, \dots, N, \quad (4)$$

$$\sum_{i=0}^N x_{it}^k - \sum_{j=0}^N x_{ij}^k = 0, \quad k = 1, 2, \dots, K; \quad t = 1, 2, \dots, N, \quad (5)$$

$$\sum_{j=0}^N q_j \left(\sum_{i=0}^N x_{ij}^k \right) \leq Q, \quad k = 1, 2, \dots, K, \quad (6)$$

$$\sum_{j=1}^N x_{0j}^k \leq 1, \quad k = 1, 2, \dots, K, \quad (7)$$

$$\sum_{i=1}^N x_{i0}^k \leq 1, \quad k = 1, 2, \dots, K, \quad (8)$$

$$x_{ij}^k \in \{0,1\}, \quad i, j = 0,1,2,\dots,N; \quad k = 1,2,\dots,K, \quad (9)$$

where K represents the number of routes (vehicle) and c_{ij}^k is the traveling cost from node v_i to v_j throughout route k . Objective function of Equation (1) is minimization of the total costs in all routes. Constraints (3) and (4) ensure that the service will be provided exactly once for every customer. Constraint (5) shows continuity of the route. Equation (6) represents that the total amount of demands in each route cannot exceed the vehicle's capacity. Equations (7) and (8) ensures that each route should not be used more than one time. Equation (9) also ensures that variables can only determine integer values of 0 or 1.

Figures (1) and (2) represent a sample inputs of a CVRP and its possible solution.



Figure 1. Inputs of a CVRP

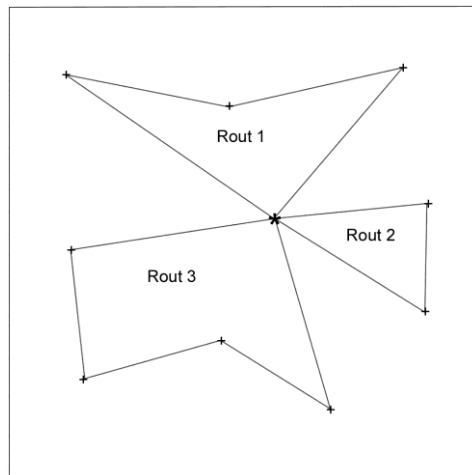


Figure 2. A possible answer for depicted problem in Figure 1 (the total demands of cities of each route is less than Q)

3. Modeling of the problem based on simulation of spider routing in the web

Like many other arthropods, spiders are able to find their route in the web through their body imbalance and sensors and also by relying on their eyes. Web maker spiders feel vibration of their webs by using their sensors and recognize the situation of caught hunt. Arachnid stimulations are sensors that report applied forces to muscle which is well

perceived by the spider through the amount of bending the body and leg joints. Spiders have narrow gaps in their joints through which feel imbalance due to bending feet.

If an insect is trapped in an extended cobweb, it will try to save itself. These moves not only do not cause its freedom, but also leads to more captivity of insect and more vibration of weaved cobwebs that makes spider aware of captured hunt. On the other hand, spiders have sharp eyes that help them find the route. Captured insects in the cobweb have different weights and vibrate the web when being caught. These forces distribute in the web surface and make the basis of spiders' decision making to achieve their hunts. But spiders should do it in the shortest possible time before their hunts to be freed or looted by other muggers. This distributed force by the insect will be extended in webs which are attached to it. But major amount of each share is dependent on the length of the web. Shorter webs have greater share in this force. Therefore, the located node on the other side of the web perceives this vibration force better. Therefore, spider attempts to provide its food in the shortest possible time through the fallen insects in the web and after resting and food digestion, it comes back to the web. Table (1) represents the comparison between the cobweb based on above explanations and CVRP problem.

Table 1. Comparison of elements used in the web algorithm with CVRP problem.

$G(V,E)$ Graph	Nodes	$s(v_i, v_j)$ edge	Nodes demand	Nodes demand	Searcher Capacity
CVRP Problem	Customers	Route Between two customers of v_i and v_j	q_i	Depot	Vehicle Capacity
Web Algorithm	Insects	Web Jointing Two nodes of v_i and v_j	Insects Weight	Spider Den	Spider Appetite

3.1 Calculation of Vibration Matrix

Searcher starts its movement to find the shortest route. Therefore, it chooses its route towards direction of a web with the most amount of vibration among webs, which are connected to the initial point. But this selection is a random and possible selection in a way that a web with the most vibration has the highest possibility. Vibration of $v(i, j)$ web is based on two factors:

Direct force distributed from v_i node, and indirect force distributed from other nodes which will be extended to $v(i, j)$ edge. In other words, the force releases from $v_h, (h \neq i, j)$ node and then passes v_i node. In order to find each force, we firstly consider distance of v_j routes with maximum amount of two edges. Routes starting from different $v_h, (h \neq i, j)$ nodes will continue to v_j node. However, only those routes that end in $v(i, j)$ web are considered here (Eq. (10) and (11)). Routes with the maximum amount of two edges are selected because they prevent complexity of calculation. Therefore, considering routes with more than two edges will increase the algorithm accuracy. In this regard, d_{hij} is the distance of a route starting from v_h which leads to v_j node after passing v_i .

$$d_{hij} = \begin{cases} d_{hi} + d_{ij} & \text{if } h \neq i, j \\ d_{ij} & \text{e.t.} \end{cases} \quad (10)$$

$$D_{hij} = \frac{1}{d_{hij}} \quad (11)$$

According to the definition of D_h as the inverse set in routes with maximum amount of two edges connected to v_h node (Eq. (12)), the force imposed on $v(i, j)$ web from v_h node can be obtained based on Eq. (13).

$$D_h = \sum_{s \neq h} \sum_{l \neq h} D_{hsl} \quad (12)$$

$$r_{hij} = (D_{hij} / D_h) \times w_h \tag{13}$$

where w_h represents the weight of v_h node. In the investigated problem in this research, all w_h coefficients are considered equal to 1. r_{hij} is the force inserted to $v(i, j)$ web resulting from v_h node. By summing all forces from different nodes on $v(i, j)$ web, the vibration force of this node can be calculated as follows:

$$v_{ij} = \sum_h r_{hij} \tag{14}$$

Final result of the above calculations of vibration matrix is $V = (v_{ij})$, in which matrix element of v_{ij} represents vibration in $v(i, j)$ edge. Now we can successfully produce a matrix that its element represents possibility of choosing each web nodes in every step of decision making process. In this step, it is necessary to use another element as better simulation of search process. As mentioned before, in addition to nervous system, the spider uses visual sense for routing in the web net. To this end, we can use visibility power matrix. In distance matrix, d_{ij} represents the web length connecting v_i and v_j nodes. The shorter the web length is, the more visibility the searcher has. Therefore, the searcher with more ability can see its next node. So $\Gamma = (\gamma_{ij})$ matrix will be defined as the visibility matrix that will be defined as Eq. (15).

$$\gamma_{ij} = 1 / d_{ij} \tag{15}$$

By combining two Γ and V matrices, we will be able to make possibility matrix which use both sense of vision and mechanical sensors of spider for identification and tracking of routing (Eq. (16)).

$$P_{ij} = \frac{v_{ij}^\alpha \times \gamma_{ij}^\beta}{\sum_j v_{ij}^\alpha \times \gamma_{ij}^\beta} \tag{16}$$

α and β coefficients in Eq. (16) represent the importance degree of mechanical sensor and spider’s visual sense in hunting and routing, respectively. Therefore, the appropriate values of these coefficients are dependent on the nature and structure of investigated problem. This matrix will be a guide for spider in recognition and routing in the cobweb.

4. Description of the web algorithm

By using the modelling in section 3, the spider will be an index for a vehicle that in each route chooses its hunts so that meet its appetite and then comes back to its den. The next route will be among nodes which have not been chosen in the previous routes. This act will be continued until there is not any insects left in the web which has not been hunted by the spider. Decision making for selecting the next nodes according to the possibility rule arises from both matrices of vibration and visibility power. Therefore, in order to choose the next nodes of v_j from k^{th} route in the v_i node, the spider will use the following possible formula:

$$pr_{ij}(k) = \begin{cases} \frac{P_{ij}}{\sum_{h \neq tabu_k} P_{ih}} & j \notin tabu_k \\ 0 & otherwise \end{cases} \tag{17}$$

where $pr_{ij}(k)$ is the possibility of choosing (v_i, v_j) web in the k^{th} route. Moreover, $tabu_k$ is a set of impossible nodes (selected nodes up to this stage) for k^{th} route.

4.1 Synchronization of the possibility matrix

In the process of extracting the optimum answer, the related area will be determined by good answer. Therefore, this area in the process is promising to find better answers and continues a comprehensive search. In order to find better answers, the synchronization of the possibility matrix will be used. When the spider looks for finding the shortest possible route, attempting to strengthen the web by secreting basic component of the web resembles the good responses. This operation by using equation of synchronization of the possibility matrix is provided as follows:

$$pr_{ij}^{new} = \rho \times r_{ij}^{old}; (v_i, v_j) \in rout_* \quad (18)$$

where pr_{ij}^{new} is vibration value in the (v_i, v_j) web after synchronization, pr_{ij}^{old} is vibration value in the (v_i, v_j) before synchronization and the coefficient ρ represents the web maker material. Also $rout_*$ represents the local optimized route in each stage of the algorithm iteration. So, the algorithm can be described as Table (2).

Table 2. Web algorithm steps description.

0. primary amount: Calculate P matrix . Put minimize value of distance equal with a large number.
1. Repeate the algorithm until establishing its stop condition. (first loop) 1.1. Repeat following steps until all nodes will be meeting by the spider. (second loop) 1.1.1. Define a new route k . 1.1.2. Start your route from the initial node. 1.1.3. Repeat following steps until Q capacity will be completed. (third loop) 1.1.3.1. Name the current node as i . 1.1.3.2. Extract the possibility of route selecting from node i by pr_{ij} vector. 1.1.3.3. Select the next non visited node by random number. 1.1.4. (end of the third loop) 1.1.5. Save in the memory the travelled route and its distance. 1.2. (end of the second loop) 1.3. If the current rout distance smaller than the minimum value, then put the best rout equal with current rout. 1.4. Increase q percent the matrix elements of pr related to selected route.
2. (end of the first loop)

5. Numerical examples

Described web algorithm in the previous section is used to find the optimum answers of three standard problems of CVRP with different dimensions, namely P-n19-k2, E-n33-k14, and B-n78-k10. The web algorithm is performed in MATLAB 7.1 software and the obtained results are shown in table (3).

Table 3. The obtained results of CVRP with different dimensions.

Problem	answer and time	performance 1	performance 2	performance 3	performance 4
P-n19-k2	answer	212	212	212	212
	time	5	6	10	4
E-n33-k14	answer	847	847	850	850
	time	21	7	3	11
B-n78-k10	answer	1288	1275	1305	1285
	time	25	71	72	58

The related information to these three problems is provided in columns 1 to 4 of Table (4), including the problem's name, the number of cities, the capacity of vehicle Q , and the best obtained optimum answer until this time. The obtained results are shown in columns 5 to 7, which represent the best and the worst answers, and the average of obtained answers. Parameters used for CVRP samples are: $\rho = .005$, $\alpha = 9$ $\beta = 2$. Although parameters are dependent to the problem's structure, the experiments show that these values are suitable for all of the examples.

Table 4. The computational results using web algorithm.

problem	N	Q	Best Known	Best	Worst	Average
P-n19-k2	19	160	212	212	212	212
E-n33-k14	33	8000	835	847	850	848
B-n78-k10	78	100	1266	1275	1305	1290

As it is visible in Table (4), the obtained results represent that the performance of the algorithm has the error value of 0%, 1%, and 1% from optimum answers, respectively. The obtained answers from web algorithm clarifies that this algorithm achieves proper answers quickly. Performing this test and the obtained results represent the accuracy of the above claims.

6. Conclusion

In this paper, a new metaheuristic algorithm, inspired by nature, is introduced. As a practical representation of this algorithm, the capacity vehicle routing problem is used in the form of an NP-Hard problem by web algorithm, in which the obtained results indicate the efficiency and capability of this algorithm in solving difficult problems. This study considers three standard problems of CVRP with various dimensions and MATLAB 7.1 software is applied for evaluating the performance of the proposed web algorithm. The obtained results show that the results of the new algorithm is one percent or less away from the optimal solution. Therefore, the developed algorithm in this research is suitable for solving large scale CVRP in a reasonable time. Although web algorithm shows its success in this test, in order to take advantage of other algorithms and find closer answers to the optimum values, we can extend web algorithm and combine it with known metaheuristic algorithms, such as genetic algorithm, by making a hybrid algorithm. The ability of the web algorithm in finding quick, proper and near optimization answers can be considered as a factor for using this algorithm as a method to find initial ideal answers in the problems with large dimensions.

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